**Poisson Distribution and Traffic Applications**

The Poisson distribution is a discrete probability distribution that applies to the number of occurrences of some event over a specified interval, usually time, but may also be some other unit (area, volume, etc.)

The random variable x is the number of occurrences of an event over the specified interval. The occurrences must be random, independent of each other, and uniformly distributed over the interval.

The probability of an event occurring x times over a given interval is: P $\left(x\right)=\frac{μ^{x}e^{-x}}{x!}$ , µ is the average number of occurrences per interval.

**Example 1:** There is an average of 2.25 births per day at a local hospital. What is the probability that on any given day there are a) no births, b) 3 births, c) at least three births?

**Example 2:** A shopping mall has an incident of shoplifting (on average) once every three hours. The mall is open from 10:00 a.m. to 9 p.m. What is the probability that during a single business day there is/are a) at least one shoplifting incident, b) no shoplifting incidents?

**Traffic Application: Vehicle Arrivals**

The number of vehicles likely to arrive at a given location during a specified time period is an application of the Poisson distribution. If λ is the average vehicle flow rate (veh/time), t is the desired time period, and n is the number of vehicles arriving during the interval then P $\left(n\right)=\frac{(λt)^{n}e^{-(λt)}}{n!}$ , where λt is the average number of arrivals during time period t.

**Transportation Example 1:** A traffic study counted 650 cars arriving at a certain intersection during rush hour. Intersection geometry allows seven cars to back up at a red light. If the light stays red for 35 seconds, what is the probability that the platoon becomes too long for the intersection?

**Traffic Application: Time Gap between Vehicle Arrivals**

Traffic engineers refer to the time gap between two moving vehicles as “headway”. Since a certain amount of time is needed to safely pull out into traffic it is important that engineers understand the distribution of headways, h in a stream of moving traffic.

During a gap in traffic there are no vehicle arrivals (n = 0). The Poisson distribution can be used to evaluate the probability of a gap time exceeding a certain value, P(h > t). If λ is the average vehicle flow rate (veh/time) and t is the gap time then λt is the average number of arrivals during time t. Using the arrival equation above $P\left(0\right)=\frac{(λt)^{0}e^{-(λt)}}{0!}$ = $e^{-(λt)}$. The probability of a gap time (headway) exceeding a given time period t is therefore $P\left(h\geq t\right)=e^{-(λt)}$. This is known as the negative exponential distribution with continuous random variable t.

**Transportation Example 2:** Data for a road shows that rush hour traffic volume is 1500 vehicles per hour (Poisson distribution). How many of the 1500 gaps are expected to be 4 seconds or longer during rush hour?

**Transportation Example 3:** A vehicle pulls out onto a highway that has a flow rate of 300 veh/hr (Poisson distributed). The driver does not look for oncoming traffic. Road conditions and vehicle speeds on the highway are such that it takes 1.7 seconds for the oncoming vehicle to stop once the breaks are applied. If driver perception/reaction time is 2.5 seconds what is the probability that the vehicle pulling out will get in an accident with an oncoming vehicle?